



22127205

**MATHEMATICS
HIGHER LEVEL
PAPER 1**

Thursday 3 May 2012 (afternoon)

2 hours

Candidate session number

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Examination code

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics HL and Further Mathematics SL information booklet** is required for this paper.
- The maximum mark for this examination paper is [120 marks].

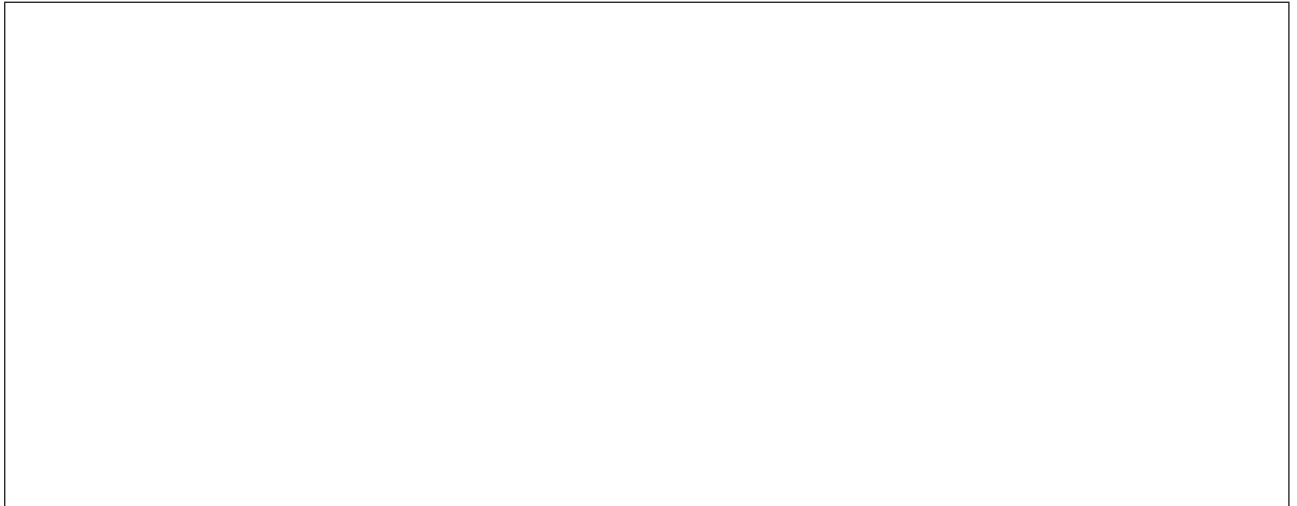


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3. [Maximum mark: 5]

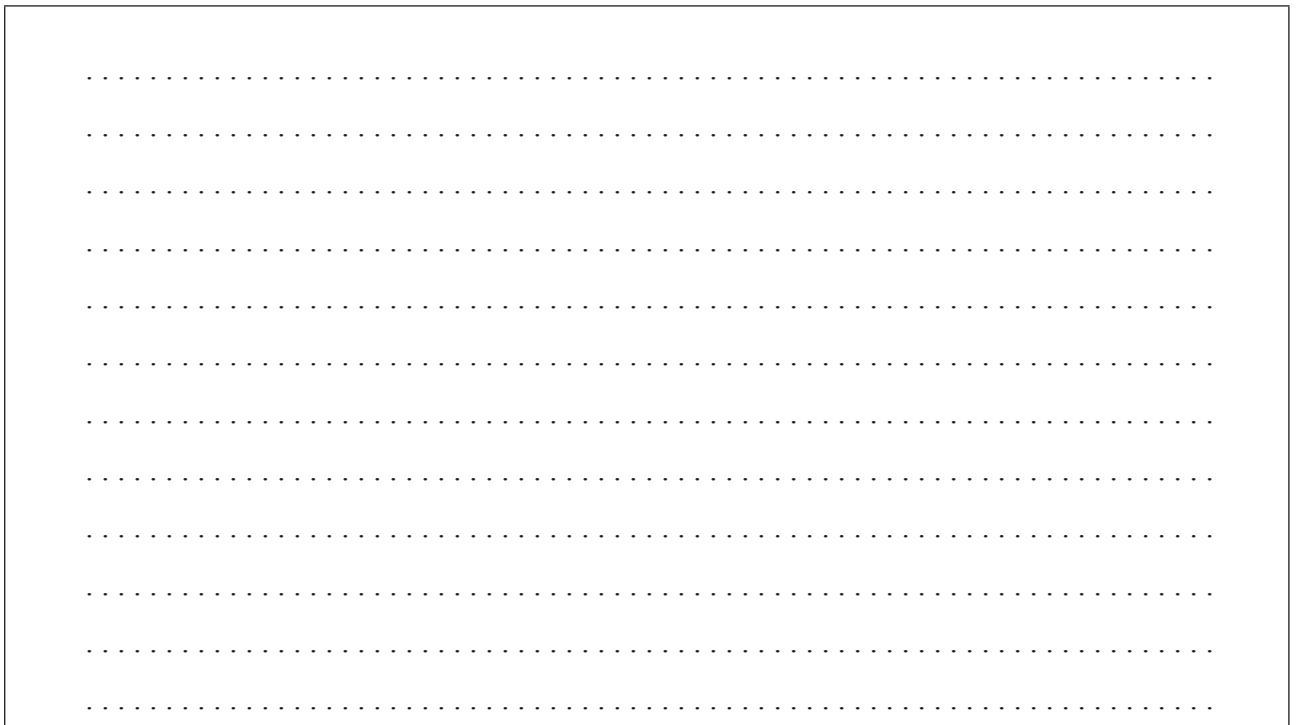
On a particular day, the probability that it rains is $\frac{2}{5}$. The probability that the “Tigers” soccer team wins on a day when it rains is $\frac{2}{7}$ and the probability that they win on a day when it does not rain is $\frac{4}{7}$.

(a) Draw a tree diagram to represent these events and their outcomes. [1 mark]



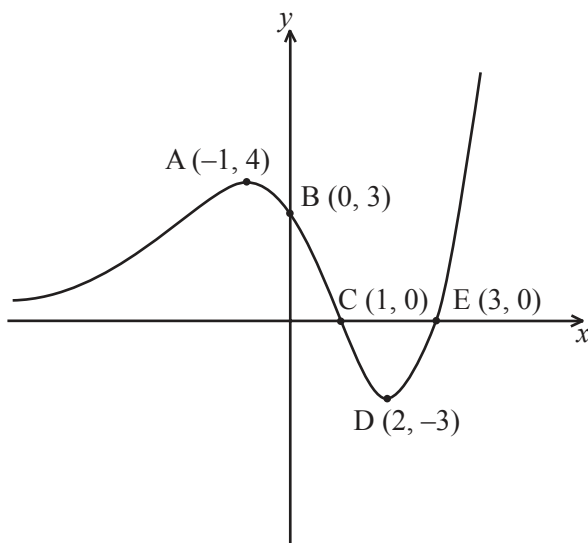
(b) What is the probability that the “Tigers” soccer team wins? [2 marks]

(c) Given that the “Tigers” soccer team won, what is the probability that it rained on that day? [2 marks]

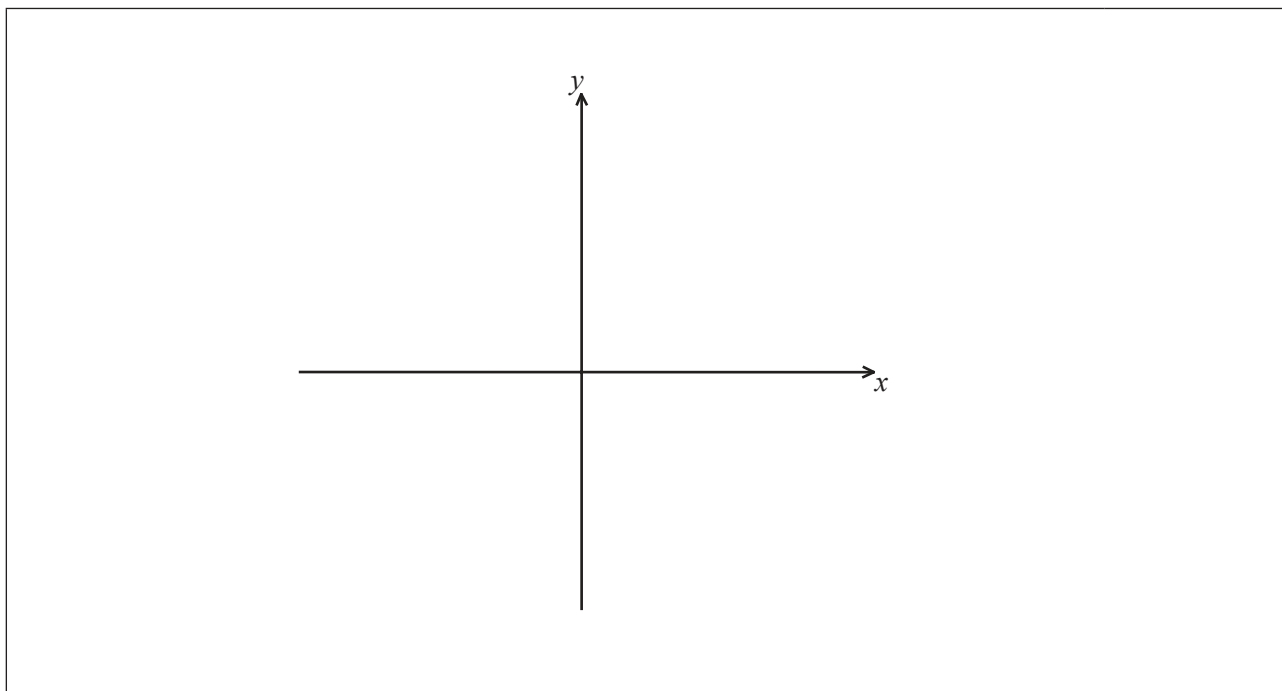


7. [Maximum mark: 6]

The graph of $y = f(x)$ is shown below, where A is a local maximum point and D is a local minimum point.



- (a) On the axes below, sketch the graph of $y = \frac{1}{f(x)}$, clearly showing the coordinates of the images of the points A, B and D, labelling them A' , B' , and D' respectively, and the equations of any vertical asymptotes. [3 marks]



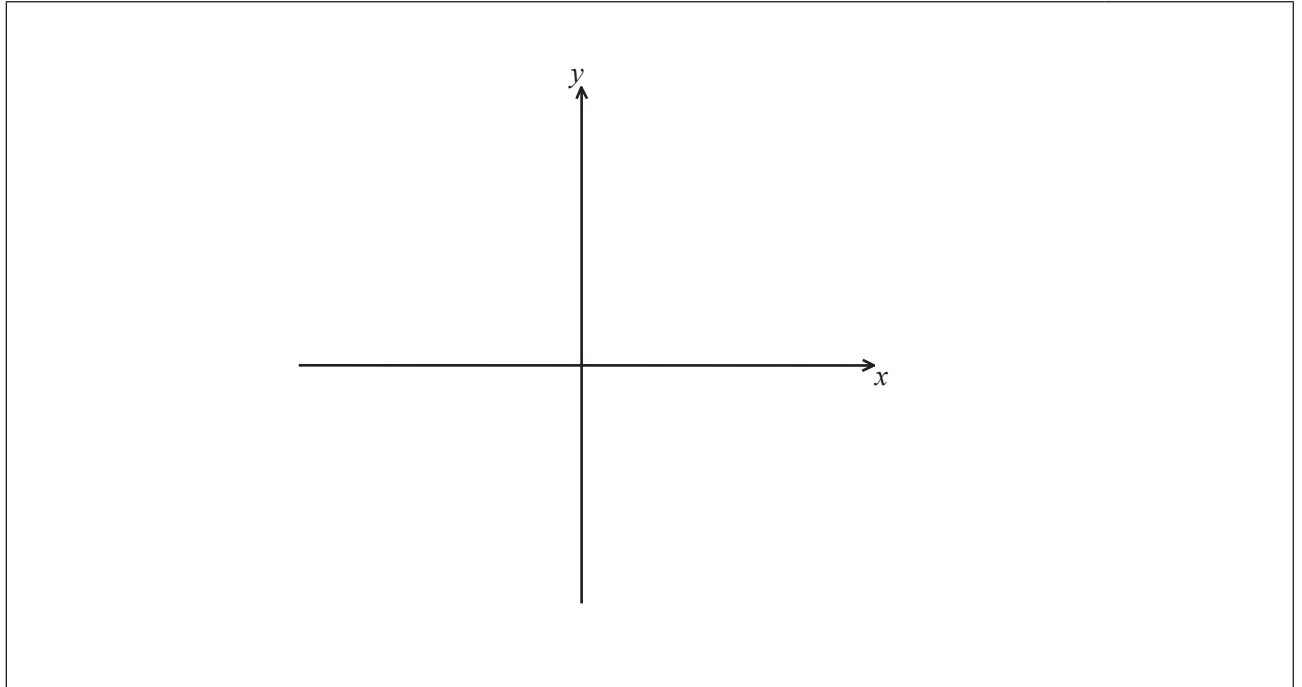
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(Question 7 continued)

- (b) On the axes below, sketch the graph of the derivative $y = f'(x)$, clearly showing the coordinates of the images of the points A and D, labelling them A'' and D'' respectively.

[3 marks]



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Turn over

10. [Maximum mark: 9]

The function f is defined on the domain $\left[0, \frac{3\pi}{2}\right]$ by $f(x) = e^{-x} \cos x$.

(a) State the two zeros of f . [1 mark]

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(b) Sketch the graph of f . [1 mark]

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Answers written on this page will
not be marked.



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SECTION B

Answer **all** questions on the answer sheets provided. Please start each question on a new page.

11. [Maximum mark: 18]

Consider the following functions:

$$f(x) = \frac{2x^2 + 3}{75}, x \geq 0$$

$$g(x) = \frac{|3x - 4|}{10}, x \in \mathbb{R}.$$

- (a) State the range of f and of g . [2 marks]
- (b) Find an expression for the composite function $f \circ g(x)$ in the form $\frac{ax^2 + bx + c}{3750}$, where a, b and $c \in \mathbb{Z}$. [4 marks]
- (c) (i) Find an expression for the inverse function $f^{-1}(x)$.
- (ii) State the domain and range of f^{-1} . [4 marks]

The domains of f and g are now restricted to $\{0, 1, 2, 3, 4\}$.

- (d) By considering the values of f and g on this new domain, determine which of f and g could be used to find a probability distribution for a discrete random variable X , stating your reasons clearly. [6 marks]
- (e) Using this probability distribution, calculate the mean of X . [2 marks]



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12. [Total mark: 29]

Part A [Maximum mark: 12]

(a) Given that $(x + iy)^2 = -5 + 12i$, $x, y \in \mathbb{R}$. Show that

(i) $x^2 - y^2 = -5$;

(ii) $xy = 6$.

[2 marks]

(b) Hence find the two square roots of $-5 + 12i$.

[5 marks]

(c) For any complex number z , show that $(z^*)^2 = (z^2)^*$.

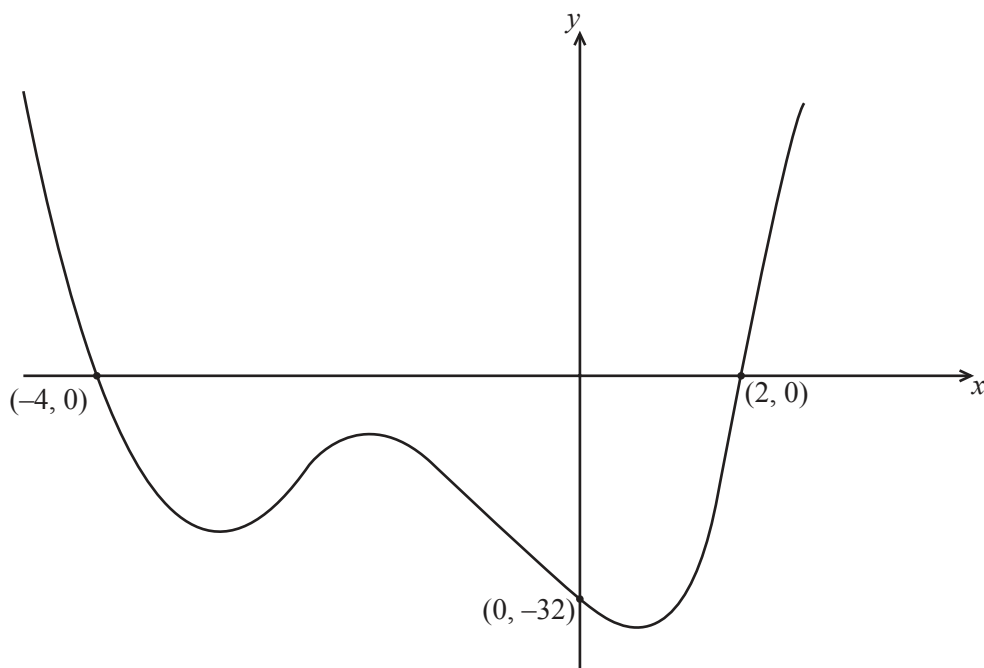
[3 marks]

(d) Hence write down the two square roots of $-5 - 12i$.

[2 marks]

Part B [Maximum mark: 17]

The graph of a polynomial function f of degree 4 is shown below.



(a) Explain why, of the four roots of the equation $f(x) = 0$, two are real and two are complex.

[2 marks]

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(Question 12 continued)

- (b) The curve passes through the point $(-1, -18)$. Find $f(x)$ in the form $f(x) = (x-a)(x-b)(x^2+cx+d)$, where $a, b, c, d \in \mathbb{Z}$. [5 marks]
- (c) Find the two complex roots of the equation $f(x) = 0$ in Cartesian form. [2 marks]
- (d) Draw the four roots on the complex plane (the Argand diagram). [2 marks]
- (e) Express each of the four roots of the equation in the form $re^{i\theta}$. [6 marks]

13. [Maximum mark: 13]

- (a) Using the definition of a derivative as $f'(x) = \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right)$, show that the derivative of $\frac{1}{2x+1}$ is $\frac{-2}{(2x+1)^2}$. [4 marks]
- (b) Prove by induction that the n^{th} derivative of $(2x+1)^{-1}$ is $(-1)^n \frac{2^n n!}{(2x+1)^{n+1}}$. [9 marks]



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